Convex Optimization Exercises - Test 1

Exercise 1. Consider the following optimization problem.

$$\begin{aligned} &\text{maximize} && f(x,y) = \min(x - y + 2, -3x + y + 6) \\ &\text{subject to} && x \geq 0, y \geq 0 \end{aligned}$$

Find an equivalent linear problem, write down its normal form, and solve it.

Exercise 2. Fix two sets $S, T \subseteq \mathbb{R}^n$ and consider the set of all points $x \in \mathbb{R}^n$ that are closer to S than to T:

$$\{x \in \mathbb{R}^n \mid \operatorname{dist}(x, S) \leq \operatorname{dist}(x, T)\}$$

where

$$\operatorname{dist}(x, S) = \inf\{\|x - y\|_2 \mid y \in S\}.$$

Prove that this set is always convex or give a counterexample. What if $S = \{x_0\}$ contains only one point?